Factors of Polynomials

ACTIVITY 17

How Many Roots?
Lesson 17-1 Algebraic Methods

My Notes

Learning Targets:

- Determine the linear factors of polynomial functions using algebraic methods.
- Determine the linear or quadratic factors of polynomials by factoring the sum or difference of two cubes and factoring by grouping.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Note Taking, Look for a Pattern, Simplify a Problem, Identify a Subtask

When you factor a polynomial, you rewrite the original polynomial as a product of two or more polynomial factors.

1. State the common factor of the terms in the polynomial $4x^3 + 2x^2 - 6x$. Then factor the polynomial.
   $2x$; $2x(2x^2 + x - 3)$

2. **Make use of structure.** Consider the expression $x^2(x - 3) + 2x(x - 3) + 3(x - 3)$.
   a. How many terms does it have? 3
   b. What factor do all the terms have in common? $x - 3$

3. Factor $x^2(x - 3) + 2x(x - 3) - 3(x - 3)$.
   $(x - 3)(x^2 + 2x - 3)$

Some quadratic trinomials, $ax^2 + bx + c$, can be factored into two binomial factors.

Example A

Factor $2x^2 + 7x - 4$.

Step 1:	Find the product of a and c.	$2(-4) = -8$
Step 2:	Find the factors of ac that have a sum of b, 7.	$8 + (-1) = 7$
Step 3:	Rewrite the polynomial, separating the linear term.	$2x^2 + 8x - 1x - 4$
Step 4:	Group the first two terms and the last two terms.	$(2x^2 + 8x) + (-x - 4)$
Step 5:	Factor each group separately.	$2x(x + 4) - 1(x + 4)$
Step 6:	Factor out the binomial.	$(x + 4)(2x - 1)$

Solution: $2x^2 + 7x - 4 = (x + 4)(2x - 1)$

MATH TIP

Check your answer to a factoring problem by multiplying the factors together to get the original polynomial.

Common Core State Standards for Activity 17

HSN-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
HSA-SSE.A.2	Use the structure of an expression to identify ways to rewrite it.

ACTIVITY 17
Directed

Activity Standards Focus

In Activity 17, students will build on techniques they have learned previously for factoring polynomials, such as factoring trinomials and factoring a difference of squares to factor higher-order polynomials. Be sure students are comfortable with the quadratic formula and concepts related to complex solutions of equations. Many students find factoring challenging. Encourage students to follow the models provided and to look for structure in each problem.

Lesson 17-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–3 Example A Example B
Check Your Understanding
#8–9 #10 #11 #12
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Give students a pair of whole numbers, such as 124 and 264. Ask them to write the factors of each number and to identify the greatest common factor. Invite students to share their strategies for finding the greatest common factor. If time allows, continue with other whole numbers, or with sets of three or four numbers.

1–3 Activating Prior Knowledge, Discussion Groups These items will help students recall the concept of a factor and then extend it to polynomials beyond binomial factors. Students may need to spend some time on these items to succeed. Have students who have factored correctly share their work. You may want to give an example with a trinomial factor as well.

Example A Activating Prior Knowledge, Note Taking In Activity 7, students factored quadratic trinomials. This example shows factoring a quadratic trinomial by grouping. It may help facilitate understanding if students make the connection back to the box method used in Lesson 7-2.

Example B Note Taking, Activating Prior Knowledge, Debriefing Remind students that they can check answers by multiplying the factors and comparing the product with the original expression.

ELL Support

Monitor students' understanding of the examples by asking key questions about the mathematical procedures or by asking them to summarize the process. Listen for clear communication of mathematical concepts. If necessary, assist students with pronunciation and/or provide translations of key terms.

My Notes

Try These A

a. Use Example A as a guide to factor $6x^2 + 19x + 10$. Show your work.

$a \cdot c = 6 \cdot 10 = 60$
Factors with a sum of 19: 15, 4: $15 + 4 = 19$; $15 \cdot 4 = 60$
Separate linear term: $6x^2 + 15x + 4x + 10$
Factor each group: $3x(2x + 5) + 2(2x + 5)$
Factor out the binomial: $(2x + 5)(3x + 2)$

Factor each trinomial. Show your work.

b. $3x^2 - 8x - 3$
$(x - 3)(3x + 1)$

c. $2x^2 + 7x + 6$
$(x + 2)(2x + 3)$

Some higher-degree polynomials can also be *factored by grouping*.

Example B

a. Factor $3x^3 + 9x^2 + 4x + 12$ by grouping.

Step 1: Group the terms. $(3x^3 + 9x^2) + (4x + 12)$
Step 2: Factor each group separately. $3x^2(x + 3) + 4(x + 3)$
Step 3: Factor out the binomial. $(x + 3)(3x^2 + 4)$
Solution: $3x^3 + 9x^2 + 4x + 12 = (x + 3)(3x^2 + 4)$

b. Factor $3x^4 + 9x^3 + 4x + 12$ by grouping.

Step 1: Group the terms. $(3x^4 + 9x^3) + (4x + 12)$
Step 2: Factor each group separately. $3x^3(x + 3) + 4(x + 3)$
Step 3: Factor out the binomial. $(x + 3)(3x^3 + 4)$
Solution: $3x^4 + 9x^3 + 4x + 12 = (x + 3)(3x^3 + 4)$

Try These B

Factor by grouping. Show your work.

a. $2x^3 + 10x^2 - 3x - 15$
$(x + 5)(2x^2 - 3)$

b. $4x^4 + 7x^3 + 4x + 7$
$(x^3 + 1)(4x + 7)$

My Notes

Check Your Understanding

4. Factor $7x^4 + 21x^3 - 14x^2$.
5. Factor $6x^2 + 11x + 4$.
6. Factor by grouping.
   a. $8x^3 - 64x^2 + x - 8$
   b. $12x^4 + 2x^3 - 30x - 5$
7. **Reason abstractly.** What is the purpose of separating the linear term in a quadratic trinomial when factoring?

A difference of two squares can be factored by using a specific pattern, $a^2 - b^2 = (a + b)(a - b)$. A *difference of two cubes* and a *sum of two cubes* also have a factoring pattern.

Difference of Cubes	Sum of Cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

8. What patterns do you notice in the formulas that appear above?
   Sample answer: For a difference of cubes, the cube root of the first term minus the cube root of the second term is multiplied by the quantity of the cube root of the first term squared plus the product of the cube root of each term plus the cube root of the second term squared. For a sum of cubes, the cube root of the first term plus the cube root of the second term is multiplied by the quantity of the cube root of the first term squared minus the product of the cube root of each term plus the cube root of the second term squared.

9. **Express regularity in repeated reasoning.** Factor each difference or sum of cubes.
   a. $x^3 - 8$
   $(x - 2)(x^2 + 2x + 4)$
   b. $x^3 + 27$
   $(x + 3)(x^2 - 3x + 9)$
   c. $8x^3 - 64$
   $(2x - 4)(4x^2 + 8x + 16)$
   d. $27 + 125x^3$
   $(3 + 5x)(9 - 15x + 25x^2)$

MATH TIP

It is a good strategy to first identify and label *a* and *b*. This makes it easier to substitute into the formula.

Some higher-degree polynomials can be factored by using the same patterns or formulas that you used when factoring quadratic binomials or trinomials.

10. Use the difference of squares formula $a^2 - b^2 = (a + b)(a - b)$ to factor $16x^4 - 25$. (It may help to write each term as a square.)
   $(4x^2 + 5)(4x^2 - 5)$

Check Your Understanding

Debrief students' answers to these items to ensure that they understand how to identify common factors and how to factor by grouping. Students should be able to explain the connection between common factors and factoring by grouping; factoring by grouping can be used when terms have a common binomial factor.

Answers

4. $7x^2(x^2 + 3x - 2)$
5. $(3x + 4)(2x + 1)$
6. a. $(8x^2 + 1)(x - 8)$
   b. $(2x^3 - 5)(6x + 1)$
7. Separate the linear term to factor by grouping.

Universal Access

Call students' attention to the general formulas for a difference of cubes and a sum of cubes. The formulas are so similar that they may look the same at first glance. Encourage students to mark their texts to indicate the differences between the formulas.

8–9 Look for a Pattern, Group Presentation, Debriefing The patterns of factoring a difference of cubes and a sum of cubes help students to factor cubic functions quickly. This will help in graphing functions and finding zeros later in this unit and through calculus.

10 Identify a Subtask, Simplify the Problem, Activating Prior Knowledge This item is an opportunity for students to see quadratic methods of factoring used with higher-degree polynomials.

11 Quickwrite, Think-Pair-Share, Debriefing Students should recognize that the trinomial was factored in part by using a pattern and a formula for a quadratic trinomial. Pair or group students carefully to facilitate sharing and monitor to be sure that students are communicating clearly and that discussion is adding to comprehension. Encourage students to take notes during these discussions to aid comprehension and to enhance listening skills.

12 Identify a Subtask, Simplify the Problem, Debriefing Help students make connections with the familiar patterns of factoring they used in Unit 2.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to factoring. Ask students to verbalize the patterns in polynomials that allow them to apply the formulas and procedures presented in this lesson.

Answers

13. a. $(5x + 6)(25x^2 - 30x + 36)$
b. $(x^2 - 3)(x^4 + 3x^2 + 9)$
14. a. $(x^2 - 11)(x^2 - 3)$
b. $(9x^2 + 25)(3x + 5)(3x - 5)$
15. The sum of the degrees of the factors is equal to the degree of the original polynomial.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 17-1 PRACTICE

16. $(3x + 1)(x - 5)$
17. $(2x + 3)(4x^2 - 6x + 9)$
18. $(x^2 + 4)(x + 3)(x - 3)$
19. $x(x + 3)(x - 3)(2x - 1)$
20. $2x + 3$; The area of a square is equal to the square of the length of one side. $4x^2 + 12x + 9 = (2x + 3)^2$.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand the factoring techniques and related concepts. Watch for students who do not factor completely. If students are struggling to identify the structure in higher-order polynomials, show them how to define new variables.

My Notes

11. Reason quantitatively. Explain the steps used to factor $2x^5 + 6x^3 - 8x$.

$2x^5 + 6x^3 - 8x$	Original expression
$= 2x(x^4 + 3x^2 - 4)$	Factor out the GCF.
$= 2x(x^2 + 4)(x^2 - 1)$	Factor the trinomial using a quadratic pattern.
$= 2x(x^2 + 4)(x + 1)(x - 1)$	Factor a difference of two squares.

12. Use the formulas for quadratic binomials and trinomials to factor each expression.

a. $x^4 + x^2 - 20$
$(x^2 + 5)(x + 2)(x - 2)$

b. $16x^4 - 81$
$(4x^2 + 9)(4x^2 - 9)$
$= (4x^2 + 9)(2x + 3)(2x - 3)$

c. $(x - 2)^4 + 10(x - 2)^2 + 9$
$((x - 2)^2 + 9)((x - 2)^2 + 1)$

Check Your Understanding

13. Factor each difference or sum of cubes.
a. $125x^3 + 216$
b. $x^6 - 27$

14. Use the formulas for factoring quadratic binomials and trinomials to factor each expression.
a. $x^4 - 14x^2 + 33$
b. $81x^4 - 625$

15. Attend to precision. A *linear factor* is a factor that has degree 1. A *quadratic factor* has degree 2. The factored expression in Item 11 has 3 linear factors and 1 quadratic factor. What is true about the degree of the factors in relation to the degree of the original expression?

LESSON 17-1 PRACTICE

Factor each expression.

16. $3x^2 - 14x - 5$

17. $8x^3 + 27$

18. $x^4 - 5x^2 - 36$

19. $2x^4 - x^3 - 18x^2 + 9x$

20. Model with mathematics. The trinomial $4x^2 + 12x + 9$ represents the area of a square. Write an expression that represents the length of one side of the square. Explain your answer.

MINI-LESSON: Factoring Higher-Level Polynomials

If students need additional help factoring higher-level polynomials, a mini-lesson is available to provide practice.

See the Teacher Resources at SpringBoard Digital for a student page for this mini-lesson.

My Notes

Learning Targets:

- Know and apply the Fundamental Theorem of Algebra.
- Write polynomial functions, given their degree and roots.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Note Taking, Graphic Organizer, Work Backward

As a consequence of the ***Fundamental Theorem of Algebra***, a polynomial $p(x)$ of degree $n \geq 0$ has exactly n linear factors, counting factors used more than once.

Example A

Find the zeros of $f(x) = 3x^3 + 2x^2 + 6x + 4$. Show that the Fundamental Theorem of Algebra is true for this function by counting the number of zeros.

Step 1: Set the function equal to 0. $\quad 3x^3 + 2x^2 + 6x + 4 = 0$

Step 2: Look for a factor common to all terms, use the quadratic trinomial formulas, or factor by grouping, as was done here. $\quad (3x^3 + 6x) + (2x^2 + 4) = 0$

Step 3: Factor each group separately. $\quad 3x(x^2 + 2) + 2(x^2 + 2) = 0$

Step 4: Factor out the binomial to write the factors. $\quad (x^2 + 2)(3x + 2) = 0$

Step 5: Use the Zero Product Property to solve for x. $\quad x^2 + 2 = 0$, $x = \pm i\sqrt{2}$ $\qquad 3x + 2 = 0$, $x = -\frac{2}{3}$

Solution: $x = \pm i\sqrt{2}$; $x = -\frac{2}{3}$.
All three zeros are in the complex number system.

Try These A

Find the zeros of the functions by factoring and using the Zero Product Property. Show that the Fundamental Theorem of Algebra is true for each function by counting the number of complex zeros.

a. $f(x) = x^3 + 9x$
$x = 0, x = \pm 3i$; 3 zeros

b. $g(x) = x^4 - 16$
$x = \pm 2i, x = \pm 2$; 4 zeros

c. $h(x) = (x - 2)^2 + 4(x - 2) + 4$
$x = 0$; 1 double zero

d. $p(x) = x^3 - 64$
$x = 4,\ x = -2 \pm 2i\sqrt{3}$; 3 zeros

e. $k(x) = x^3 + 5x^2 + 9x + 45$
$x = -5, x = \pm 3i$; 3 zeros

f. $w(x) = x^3 + 216$
$x = -6, x = 3 \pm 3i\sqrt{3}$; 3 zeros

MATH TERMS

Let $p(x)$ be a polynomial function of degree n, where $n > 0$. The **Fundamental Theorem of Algebra** states that $p(x) = 0$ has at least one zero in the complex number system.

MATH TIP

When counting the number of zeros, remember that when solutions have the $\pm$ symbol, such as $\pm a$, this represents two different zeros, a and $-a$.

MATH TIP

All real numbers are complex numbers with an imaginary part of zero.

MATH TIP

When you factor the sum or difference of cubes, the result is a linear factor and a quadratic factor. To find the zeros of the quadratic factor, use the quadratic formula.

Lesson 17-2

PLAN

Pacing: 1 class period
Chunking the Lesson
#1–5 Example A
Check Your Understanding
#7 Example B Example C
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Challenge students with a timed factoring drill. Present students with several trinomials and ask them to factor as many as they can in five minutes. Discuss the strategies students used to help them factor quickly and efficiently.

Teacher to Teacher

In higher-level courses, the statement of the Fundamental Theorem of Algebra is "Every polynomial $P(x)$ of degree $n \geq 1$ with complex coefficients has at least one root, which is a complex number (real or imaginary)." In Algebra 2, students do not work with imaginary coefficients.

Example A Note Taking, Activating Prior Knowledge, Debriefing
Students will need to use the quadratic formula for some of the Try These items. It may be necessary to remind some students of the formula. Each problem will factor initially but may not be completely factorable over the integers.

Technology Tip

On the graph of a polynomial function, the real zeros occur where the graph crosses the x-axis, while the imaginary zeros do not appear anywhere on the graph. If a polynomial function is of degree n and its graph crosses the x-axis n times, then all of the zeros are real (and distinct). If the graph does not cross the x-axis, then all of the zeros are imaginary. If the graph crosses the x-axis but does so fewer than n times, then there are several possibilities. There could be a zero with multiplicity greater than 1, or there could be imaginary zeros, or perhaps both. Students can graph the polynomial functions in this lesson on graphing calculators to explore these relationships.

For additional technology resources, visit SpringBoard Digital.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to the Fundamental Theorem of Algebra. Discuss with students the difference between a polynomial expression and a polynomial function. Invite students to describe the relationships among the degree of a polynomial, the linear factors of the polynomial, and the zeros of the corresponding polynomial function.

Answers

1. $\pm 3i, \pm 3$, 4 zeros
2. $-2, 1 \pm \sqrt{3}i$, 3 zeros
3. 0 (double zero), $\pm 5i$, 4 zeros
4. $7, \pm 2i$, 3 zeros
5. 5 factors
6. Complex zeros occur in pairs, so there must be at least 1 real zero.

7 Graphic Organizer, Group Presentation, Debriefing Students will have many different ways to illustrate the answer. Ask several students to present their answers to this item. Monitor presentations to ensure that students are using math terms and other terms correctly.

Example B Note Taking, Work Backward Note that there are other polynomial functions with these zeros. For example, multiplying the polynomial by any constant will result in a different polynomial function with the same zeros.

ACTIVITY 17
continued

My Notes

Lesson 17-2
The Fundamental Theorem of Algebra

Check Your Understanding

For Items 1–4, find the zeros of the functions. Show that the Fundamental Theorem of Algebra is true for each function by counting the number of complex zeros.

1. $g(x) = x^4 - 81$
2. $h(x) = x^3 + 8$
3. $f(x) = x^4 + 25x^2$
4. $k(x) = x^3 - 7x^2 + 4x - 28$
5. **Make use of structure.** As a consequence of the Fundamental Theorem of Algebra, how many linear factors, including multiple factors, does the function $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ have?
6. What is the minimum number of real zeros for the function in Item 4? Explain your reasoning.

7. Create a flowchart, other organizational scheme, or set of directions for finding the zeros of polynomials.
Answers will vary. Check students' work.

It is possible to find a polynomial function, given its zeros.

MATH TIP

If a is a zero of a polynomial function, then $(x - a)$ is a factor of the polynomial.

Example B

Find a polynomial function of 3rd degree that has zeros 0, 2, and -3.

Step 1: Write the factors. $f(x) = (x)(x - 2)(x + 3)$

Step 2: Multiply the binomials. $f(x) = (x)(x^2 + x - 6)$

Step 3: Distribute the x. $f(x) = x^3 + x^2 - 6x$

Solution: $f(x) = x^3 + x^2 - 6x$

Try These B

Find a polynomial function with the indicated degree and zeros.

a. $n = 3$; zeros 0, 5, -7
$f(x) = x^3 + 2x^2 - 35x$

b. $n = 4$; zeros ± 1, ± 5
$f(x) = x^4 - 26x^2 + 25$

Lesson 17-2
The Fundamental Theorem of Algebra

My Notes

The *Complex Conjugate Root Theorem* states that if $a + bi$, $b \neq 0$, is a zero of a polynomial function with real coefficients, the conjugate $a - bi$ is also a zero of the function.

Example C

a. Find a polynomial function of 3rd degree that has zeros 3 and $4i$.

Step 1: Use the Complex Conjugate Root Theorem to find all zeros. $x = 3, x = 4i, x = -4i$

Step 2: Write the factors. $f(x) = (x - 3)(x - 4i)(x + 4i)$

Step 3: Multiply the factors that contain i. $f(x) = (x - 3)(x^2 + 16)$

Step 4: Multiply out the factors to get the polynomial function. $f(x) = x^3 - 3x^2 + 16x - 48$

Solution: $f(x) = x^3 - 3x^2 + 16x - 48$

b. Find a polynomial function of 4th degree that has zeros 1, -1, and $1 + 2i$.

Step 1: Use the Complex Conjugate Root Theorem to find all zeros. $x = 1, x = -1, x = 1 + 2i, x = 1 - 2i$

Step 2: Write the factors. $f(x) = (x - 1)(x + 1)$
$(x - (1 + 2i))(x - (1 - 2i))$

Step 3: Multiply using the fact that $(a - b)(a + b) = a^2 - b^2$. $f(x) = (x^2 - 1)(x^2 - 2x + 5)$

Step 4: Multiply out the factors to get the polynomial function. $f(x) = x^4 - 2x^3 + 4x^2 + 2x - 5$

Solution: $f(x) = x^4 - 2x^3 + 4x^2 + 2x - 5$

Try These C

Reason quantitatively. Write a polynomial function of nth degree that has the given real or complex roots.

a. $n = 3$; $x = -2$, $x = 3i$
$f(x) = x^3 + 2x^2 + 9x + 18$

b. $n = 4$; $x = 3$, $x = -3$, $x = 1 + 2i$
$f(x) = x^4 - 2x^3 - 4x^2 + 18x - 45$

c. $n = 4$; $x = 2$, $x = -5$, and $x = -4$ is a double root
$f(x) = x^4 + 11x^3 + 30x^2 - 32x - 160$

Differentiating Instruction

Support students who have difficulty making connections by relating the Complex Conjugate Root Theorem to the quadratic formula. The quadratic formula yields two solutions, one adding the square root term and one subtracting it, unless the discriminant is equal to 0. If the formula yields complex solutions, they are of the form $a \pm bi$, where a and b are real numbers.

Example C Note Taking, Work Backward, Debriefing Have students explain why they should find the products of the real factors and the complex factors separately instead of attempting to multiply factors from left to right. Notice in Try These C part c the mention of a double zero. Be sure that students write the factor $(x + 4)$ twice.

Teacher to Teacher

A root b of a polynomial is called a double root, or a root of multiplicity 2, of the polynomial if $(x - b)$ appears as a factor of the polynomial exactly twice. In general, if r is a root of a polynomial and $(x - r)$ appears as a factor of the polynomial exactly k times, then the root r is called a "root of multiplicity k."

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to the Complex Conjugate Root Theorem. For Items 9–12, challenge students to find more than one possible answer.

Answers

8. $x = 3 - 2i$
9. $f(x) = x^3 - 3x^2 - 10x$
10. $f(x) = x^3 - 3x^2 + 4x - 12$
11. $f(x) = x^4 - 24x^2 - 25$
12. $f(x) = x^4 - 3x^3 - 12x^2 + 52x - 48$

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 17-2 PRACTICE

13. $x = -10, 5 \pm 5i\sqrt{3}$; 3 zeros
14. $x = 4, x = 5i, x = -5i$; 3 zeros
15. $x = 0, x = 3, x = i, x = -i$; 4 zeros
16. $f(x) = x^3 - 9x^2 + 4x - 36$
17. $f(x) = x^3 - 7x^2 + 9x + 17$
18. $f(x) = x^4 + 8x^3 - 8x^2 - 96x + 144$
19. $n = 5$. Sample answer: Use the Complex Conjugate Root Theorem to find the remaining zeros, $-2i$ and $4 - i$. There are a total of five zeros. Now use the Fundamental Theorem of Algebra to determine that the degree is 5.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand the basic concepts of the Fundamental Theorem of Algebra and the Complex Conjugate Root Theorem. If students require more practice, organize them into pairs or small groups. Each student creates a list of zeros and then writes a polynomial function that has those zeros. The students exchange polynomials and work backward to identify the zeros. You may choose to add guidelines to ensure students include double zeros and complex zeros in their constructions.

My Notes

Check Your Understanding

8. Reason abstractly. If $3 + 2i$ is a zero of $p(x)$, what is another zero of $p(x)$?

For Items 9–12, write a polynomial function of nth degree that has the given real or complex roots.

9. $n = 3; x = -2, x = 0, x = 5$
10. $n = 3; x = 3, x = 2i$
11. $n = 4; x = 5, x = -5, x = i$
12. $n = 4; x = 3, x = -4$, and $x = 2$ is a double root

LESSON 17-2 PRACTICE

For Items 13–15, find the zeros of the functions. Show that the Fundamental Theorem of Algebra is true for each function by counting the number of complex zeros.

13. $f(x) = x^3 + 1000$
14. $f(x) = x^3 - 4x^2 + 25x - 100$
15. $f(x) = x^4 - 3x^3 + x^2 - 3x$

For Items 16–19, write a polynomial function of nth degree that has the given real or complex zeros.

16. $n = 3; x = 9, x = 2i$
17. $n = 3; x = -1, x = 4 + i$
18. $n = 4; x = -6$ is a double zero and $x = 2$ is a double zero
19. Construct viable arguments. Use the theorems you have learned in this lesson to determine the degree of a polynomial that has zeros $x = 3, x = 2i$, and $x = 4 + i$. Justify your answer.

Factors of Polynomials
How Many Roots?

ACTIVITY 17 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 17-1

1. State the common factor of the terms in the polynomial $5x^3 + 30x^2 - 10x$. Then factor the polynomial.
2. Which of the following is one of the factors of the polynomial $15x^2 - x - 2$?
   A. $x - 2$
   B. $5x - 2$
   C. $5x + 1$
   D. $3x - 1$
3. Factor each polynomial.
   a. $6x^2 + 7x - 5$
   b. $14x^2 + 25x + 6$
4. Factor by grouping.
   a. $8x^3 - 64x^2 + x - 8$
   b. $12x^4 + 2x^3 - 30x - 5$
5. Factor each difference or sum of cubes.
   a. $125x^3 + 216$
   b. $x^6 - 27$
6. Use the formulas for factoring quadratic binomials and trinomials to factor each expression.
   a. $x^4 - 14x^2 + 33$
   b. $81x^4 - 625$
   c. $x^4 + 17x^2 + 60$
   d. $x^6 - 100$

Lesson 17-2

7. Which theorem states that a polynomial of degree n has exactly n linear factors, counting multiple factors?
   A. Binomial Theorem
   B. Quadratic Formula
   C. Fundamental Theorem of Algebra
   D. Complex Conjugate Root Theorem
8. Find the zeros of the functions by factoring and using the Zero Product Property. Identify any multiple zeros.
   a. $f(x) = 2x^4 + 18x^2$
   b. $g(x) = 3x^3 - 3$
   c. $h(x) = 5x^3 - 6x^2 - 45x + 54$
   d. $h(x) = 3x^4 - 36x^3 + 108x^2$
9. The table of values shows coordinate pairs on the graph of $f(x)$. Which of the following could be $f(x)$?
   A. $x(x + 1)(x - 1)$
   B. $(x - 1)(x + 1)(x - 3)$
   C. $(x + 1)^2(x + 3)$
   D. $(x + 1)(x - 2)^2$

x	$f(x)$
-1	0
0	3
1	0
2	-3

10. Write a polynomial function of nth degree that has the given zeros.
    a. $n = 3$; $x = 1, x = 6, x = -6$
    b. $n = 4$; $x = -3, x = 3, x = 0, x = 4$

ACTIVITY PRACTICE

1. $5x$; $5x(x^2 + 6x - 2)$
2. B
3. **a.** $(3x + 5)(2x - 1)$
   b. $(7x + 2)(2x + 3)$
4. **a.** $(x - 8)(8x^2 + 1)$
   b. $(6x + 1)(2x^3 - 5)$
5. **a.** $(5x + 6)(25x^2 - 30x + 36)$
   b. $(x^2 - 3)(x^4 + 3x^2 + 9)$
6. **a.** $(x^2 - 3)(x^2 - 11)$
   b. $(3x + 5)(3x - 5)(9x^2 + 25)$
   c. $(x^2 + 5)(x^2 + 12)$
   d. $(x^3 + 10)(x^3 - 10)$
7. C
8. **a.** $x = \pm 3i$, $x = 0$ (double)
   b. $x = 1$, $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
   c. $x = \pm 3$, $x = \frac{6}{5}$
   d. $x = 0$ (double), $x = 6$ (double)
9. B
10. **a.** $f(x) = x^3 - x^2 - 36x + 36$
    b. $f(x) = x^4 - 4x^3 - 9x^2 + 36x$

11. B

12. a. $f(x) = x^3 + 2x^2 - 25x - 50$
 b. $f(x) = x^4 + 16x^2 - 225$
 c. $f(x) = x^3 + x + 10$

13. a. $n = 4$
 b. $n = 5$
 c. $n = 4$

14. A

15. Sample answer: The polynomial in II is a fifth-degree polynomial; if you multiply the factors in III together, the constant term will not equal 48.

16. $f(x) = (x - 4)^2(x - i)(x + i)$; $f(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

11. Which of the following polynomial functions has multiple roots at $x = 0$?
A. $f(x) = x^2 - x$
B. $f(x) = x^3 - x^2$
C. $f(x) = x^3 - x$
D. all of the above

12. Write a polynomial function of nth degree that has the given real or complex roots.
a. $n = 3$; $x = -2$, $x = 5$, $x = -5$
b. $n = 4$; $x = -3$, $x = 3$, $x = 5i$
c. $n = 3$; $x = -2$, $x = 1 + 2i$

13. Give the degree of the polynomial function with the given real or complex roots.
a. $x = -7$, $x = 1$, $x = 4i$
b. $x = -2$, $x = 2$, $x = 0$, $x = 4 + i$
c. $x = 2i$, $x = 1 - 3i$

14. Which of the following could be the factored form of the polynomial function $f(x) = x^4 + \ldots + 48$?
I. $f(x) = (x + 1)(x + 3)(x + 4i)(x - 4i)$
II. $f(x) = (x + 2)^2(x - 1)(x + 4)(x - 6)$
III. $f(x) = (x + 3)(x - 8)(x + 2i)(x - 2i)$
A. I only
B. I and II only
C. II only
D. I, II, and III

15. Explain your reason(s) for eliminating each of the polynomials you did not choose in Item 14.

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

16. Use the information below to write a polynomial function, first in factored form and then in standard form.

Fact: The graph only touches the x-axis at a double zero; it does not cross through the axis.

Clue: One of the factors of the polynomial is $(x + i)$.